is obtained directly from Maxwell's equations.[14] Gaussian units are used and $\vec{H}$, $\vec{B}$, $\vec{E}$, $\vec{D}$, and $\vec{J}$ are the usual field quantities of electromagnetic theory. This expression assumes only that current changes are sufficiently quasistatic so that negligible energy is lost from the system by radiation. Nothing is assumed about linearity, reversibility, etc. in the magnetic material. The three terms are, respectively, the magnetic work, the electric work, and the work done in creating Joule heat by the true currents in the system. If the ferromagnet is nonconductive and incapable of storing electric energy, then only the first term

$$\delta W = \frac{1}{4\pi}\int \vec{H}\cdot\delta\vec{B}\, dV \tag{2.2}$$

is important.

The work done on the ferromagnet is stored in various forms of energy or dissipated in irreversible processes. The work expression in this form does not show this partitioning. To proceed further, the magnetic field intensity will be separated into two fields,

$$\vec{H} = \vec{H}_e + \vec{H}_d. \tag{2.3}$$

This is possible through a theorem due to Helmholtz.[15] $\vec{H}_e$ is solenoidal and is the particular solution of the equation

$$\vec{\nabla}\times\vec{H}_e = \frac{4\pi}{c}\vec{J}$$

and $\vec{H}_d$ is irrotational and is the particular solution of the equation

$$\vec{\nabla}\cdot\vec{H}_d = -4\pi\vec{\nabla}\cdot\vec{M}.$$

In other words, $\vec{H}_e$ has as sources current carrying conductors such as would be used to magnetize the magnetic material and will hereafter be called the external field. $\vec{H}_d$ has as sources surface and volume magnetic poles and will be called the demagnetizing or dipolar field. It should be remembered that an entirely equivalent partitioning can be done with the magnetic induction $\vec{B}$. The development would then evolve around the concept of free currents and Amperian currents. Although either method is acceptable, the first is commonly used since it allows greater mathematical simplicity and some physical insight depending on one's prejudices on magnetic pole concepts.

With this separation of the magnetic field intensity and with

$$\delta\vec{B} = \delta\vec{H} + 4\pi\delta\vec{M},$$

Equation (2.2) becomes

$$\delta W = \int \left[\vec{H}\cdot\delta\vec{M} + \delta\frac{H_e^2}{8\pi} + \delta\frac{H_d^2}{8\pi} + \delta\frac{\vec{H}_e\cdot\vec{H}_d}{4\pi} \right] dV.$$

In the last term, $\vec{H}_d$ is irrotational and can be written as the gradient of a scalar potential, ϕ_m. Integration by parts produces two terms. One contains $\vec{\nabla}\cdot\vec{H}_e$ and is, therefore, zero and the other contains the total divergence of $\phi_m\vec{H}_e$ and, therefore, transforms to a surface integral. Assuming the system is localized so that $r\phi_m$ and r^2H_e are regular at infinity demands that the integrand diminishes sufficiently fast so that the surface integral must vanish. The other terms can be identified. The second term is the work done in changing the external field energy and does not depend on the magnetic material. This term will be excluded from thermodynamic consideration of the ferromagnet. It is entirely a matter of bookkeeping and does not create any problems.[16] The third term is the magnetostatic "self energy" of the ferromagnet. It represents the energy required to